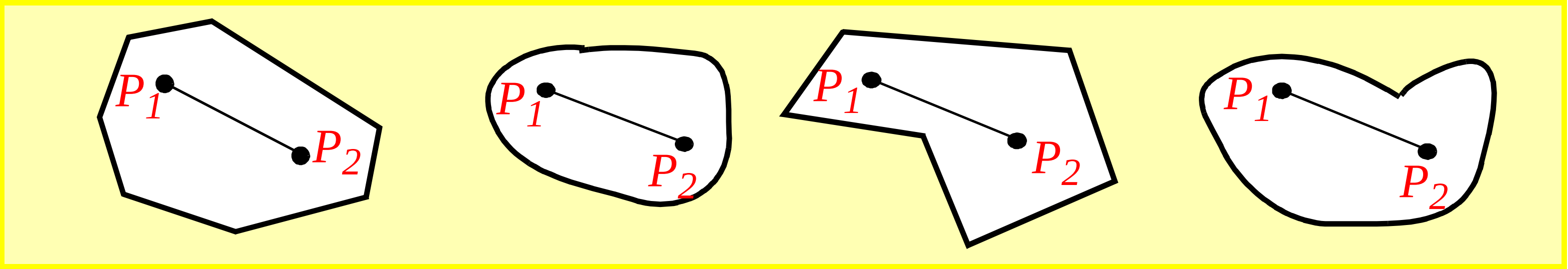
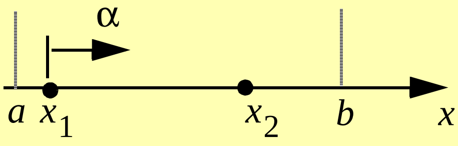
GLOBAL OPTIMALITY

- Convex Sets:
  - Let P_1 and P_2 be any two points in a set S. The set is convex if the entire line segment between the two points also remains in the set.



  - Mathematical representation of a line segment between two points

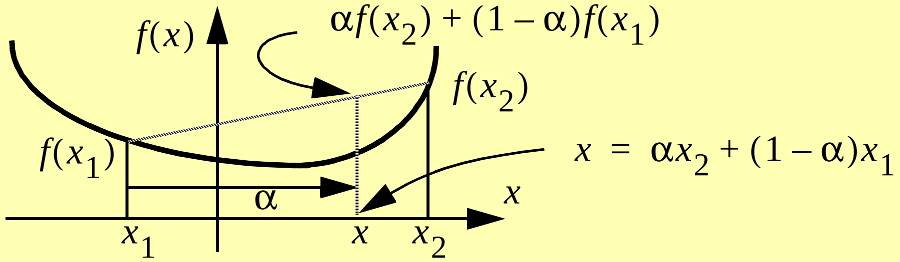


$$0 \le \alpha \le 1$$

1-D $\quad x = \alpha x_2 + (1-\alpha)x_1$

n-D $\quad \boldsymbol{x} = \alpha \boldsymbol{x}^{(2)} + (1-\alpha)\boldsymbol{x}^{(1)}$

- Convex Functions:
  - A convex function *f(x)* is defined on a convex set



  - A function *f(x)* is called convex if it lies below the line joining any two points on the curve *f(x)*

$$f(x) \le \alpha f(x_2) + (1-\alpha) f(x_1) \qquad f(\alpha x_2 + (1-\alpha)x_1) \le \alpha f(x_2) + (1-\alpha) f(x_1)$$

n-variables $$f(\alpha \mathbf{x}^{(2)} + (1-\alpha)\mathbf{x}^{(1)}) \le \alpha f(\mathbf{x}^{(2)}) + (1-\alpha) f(\mathbf{x}^{(1)})$$

  - A function of *n* variables defined on a convex set *S* is convex *if and only if* the Hessian of the function is *positive semidefinite or positive definite* at all points in the set *S*.

- Convex Programming problem:
  - If a function $g_i(\mathbf{x})$ is convex, then the set $g_i(\mathbf{x}) \leq e_i$ is convex
  - A linear equality of inequality constraint always defines a convex region
  - A nonlinear equality constraint always defines a a nonconvex feasible reg.
  - If all the equality constraints are linear and if all the inequalities written in the standard form are convex, then the feasible region is convex
  - If the cost function is convex over a convex feasible region, the problem is called to be a convex programming problem
  - For a convex programming problem, the Kuhn-Tucker first-order necessary conditions are also sufficient, and any local minimum is also a global minimum.
  - Nonconvex problems can also have global minimum points.